Block Compressed Sensing For Feedback Reduction in Relay-Aided Multiuser Full Duplex Networks

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Abstract

We propose an opportunistic full-duplex feedback algorithm that jointly estimate the feedback signal and the loop interference at the relay. We cast the problem of joint user signal-to-noise ratio (SNR) and the relay loop interference estimation at the base-station as a block sparse signal recovery problem in compressive sensing (CS). Using existing CS block recovery algorithms, the identity of the strong users is obtained and their corresponding SNRs are estimated.

1. System Description

1.1 Downlink Model

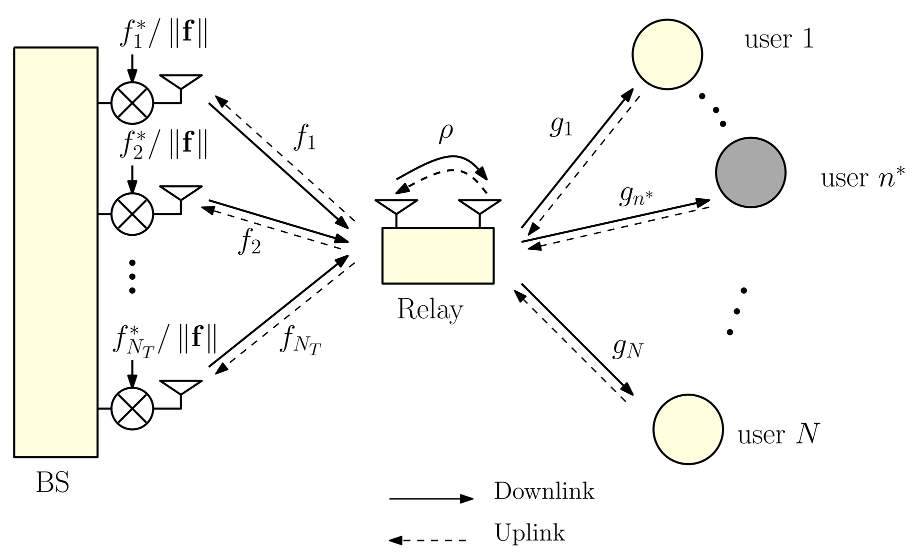

Figure 1: *Network Model.*

$$p_{|f|^2}(x) = \theta^{\theta}\frac{x^{\theta-1}}{\Gamma(\theta)}\exp(-\theta x),\ x \geq 0, \quad (1)$$

$$p_{|g|^2}(x) = \exp(-x),\ x \geq 0. \quad (2)$$

$$n^* = \arg\max_n \gamma_n,\quad \gamma_n = \frac{P_r|g_n|^2}{N_0} \quad (3)$$

$\rho \in (0,1)$ is the lopp interference at the relay.

1.2 Uplink Model (Feedback)

$\mathbf{x} = [x_1, x_2, \cdots, x_N]^t$ is the feedback vector

$$x_n = \begin{cases} \gamma_n & ,\gamma_n > \gamma_{th}. \\ 0 & ,\text{otherwise.} \end{cases} \quad (4)$$

The feedback codewords are extracted from the columns of $\mathbf{\Phi}$. The received feedback signal at the tth mini-slot at the relay is given by

$$y_r(1) = \phi_1\mathbf{x} + z_r(1). \quad (5)$$

$$y_r(t) = \phi_t\mathbf{x} + \rho y_r(t-1) + z_r(t),\ t = 2,\ldots,M, \quad (6)$$

$$\rho^k \simeq 0, \forall k \geq K.$$

The truncated feedback signal is

$$y_r(t) = \sum_{k=0}^{\min(K-1,t-1)} \rho^k\phi_{t-k}\mathbf{x} + \sum_{k=0}^{\min(K-1,t-1)} \rho^k z_r(t-k), \quad (7)$$

where the term $\kappa = \lceil \frac{K}{2} \rceil$.

After multiplying by $\frac{f_i^*}{\|\mathbf{f}\|}$, the ith antenna receives at the tth time slot

$$y_{s,i}(t) = \frac{|f_i|^2}{\|\mathbf{f}\|}\sum_{k=0}^{\min(K-1,t-1)} \rho^k\phi_{t-k}\mathbf{x} + \frac{|f_i|^2}{\|\mathbf{f}\|}\sum_{k=0}^{\min(\kappa-1,t-1)} \rho^k z_r(t-k) + \frac{f_i^*}{\|\mathbf{f}\|}w_i(t), \quad (8)$$

$$\begin{aligned} y_s(t) &= \sum_{k=0}^{\min(K-1,t-1)} \rho^k\phi_{t-k}\mathbf{x} + \sum_{k=0}^{\min(\kappa-1,t-1)} \rho^k z_r(t-k) + \sum_{i=1}^{N_T}\frac{f_i^*}{\|\mathbf{f}\|^2}w_i(t) \\ &= \sum_{k=0}^{\min(K-1,t-1)} \rho^k\phi_{t-k}\mathbf{x} + \sum_{k=0}^{\min(\kappa-1,t-1)} \rho^k z_r(t-k) + \sum_{i=1}^{N_T}\frac{f_i^*}{\|\mathbf{f}\|^2}w_i(t). \end{aligned} \quad (9)$$

2. Joint Feedback and Loop Interference Estimation

2.1 User ID Estimation Phase

We exploit the structure in (9) to cast the problem as a block CS recovery.

$$\phi_{c,n}^j \triangleq \left[\mathbf{0}_{1\times j}, \phi_{c,n}(1:M-j)\right]^t.$$

$$\mathbf{\Psi}_{(n)} \triangleq \left[\phi_{c,n}^0, \phi_{c,n}^1, \ldots, \phi_{c,n}^{K-1}\right], n = 1, 2, \ldots, N,$$

Then, the received signal in (9) can be rewritten in a matrix form as

$$\mathbf{y_s} = \mathbf{\Psi}\chi + \mathbf{z}, \quad (10)$$

where $\mathbf{\Psi} = \left[\mathbf{\Psi}_{(1)}, \mathbf{\Psi}_{(2)}, \ldots, \mathbf{\Psi}_{(N)}\right]$, $\chi = \mathbf{x} \otimes \left[1, \rho, \cdots, \rho^{K-1}\right]^t$,

$$z_t = \sum_{k=0}^{\min(\kappa-1,t-1)} \rho^k z_r(t-k) + \sum_{i=1}^{N_T} h_i w_i(t),$$

$$t = 1, \cdots, M,$$

with $h_i = \frac{f_i^*}{\|\mathbf{f}\|^2}$, $i = 1, \cdots, N_T$.

2.2 Joint SNR and Loop Interference Estimation

The BS have an estimate of the support of χ denoted by $\mathcal{J}$, where $|\mathcal{J}| = KS$. This allows to rewrite the linear system in (10) as

$$\mathbf{y}_s = \mathbf{\Psi}_{\mathcal{J}}\chi_{\mathcal{J}} + \mathbf{z}, \quad (11)$$

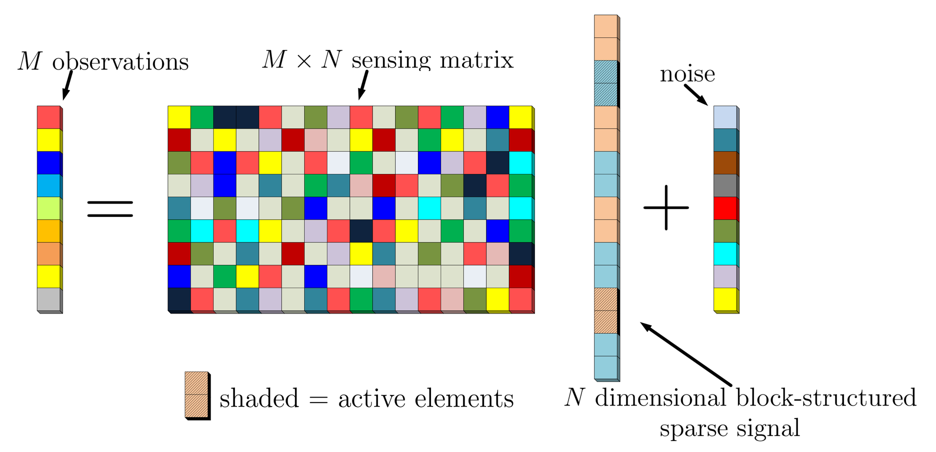

Figure 2: *Ergodic rate and Feedback load as a function of the number of users N.*

We estimate $\chi_{\mathcal{J}}$ as follows

$$\begin{aligned} \widehat{\chi}_{\mathcal{J}} &= \left(\mathbf{\Psi}_{\mathcal{J}}^t\mathbf{\Sigma}_{\mathbf{z}}^{-1}(\rho)\,\mathbf{\Psi}_{\mathcal{J}}\right)^{-1}\mathbf{\Psi}_{\mathcal{J}}^t\mathbf{\Sigma}_{\mathbf{z}}^{-1}(\rho)\,\mathbf{y}_s \\ &= \chi_{\mathcal{J}} + \epsilon, \end{aligned} \quad (12)$$

where ϵ is the estimation error vector after applying the BLUE. Also, notice that $\widehat{\chi}_{\mathcal{J}}$ has the same structure as $\chi_{\mathcal{J}}$. More precisely,

$$\widehat{\chi}_{\mathcal{J}} = \widehat{\mathbf{x}}_{\mathcal{S}} \otimes \left[1, \rho, \cdots, \rho^{K-1}\right]^t. \quad (13)$$

Using the relation in (13), it is easy to notice that the first entry of $\widehat{\mathbf{x}}_{\mathcal{S}}$ is interference free, thus we can estimate it and use it to estimate ρ subsequently.

$$\widehat{\rho} = \left[\frac{\widehat{\chi}_{\mathcal{J}}((i-1)K+j+1)}{\widehat{\mathbf{x}}_{\mathcal{S}}(i)}\right]^{1/j},\quad j = 1, \cdots, K-1,$$

$$i = 1, \cdots, S.$$

which means that we have $(K-1)S$ estimate of ρ. Averaging over all the estimates, we have the following estimate of ρ

$$\widehat{\rho} = \frac{1}{(K-1)S}\sum_{i=1}^{S}\sum_{j=1}^{K-1}\left[\frac{\widehat{\chi}_{\mathcal{J}}((i-1)K+j+1)}{\widehat{\mathbf{x}}_{\mathcal{S}}(i)}\right]^{1/j}. \quad (14)$$

2.3 BLUE Error Analysis

Denote by $\mathbf{R}_\epsilon$, the covariance matrix of ϵ in (12), then by basic manipulations, we can show that

$$\mathbf{R}_\epsilon = \left(\mathbf{\Psi}_{\mathcal{J}}^t\mathbf{\Sigma}_{\mathbf{z}}^{-1}(\rho)\,\mathbf{\Psi}_{\mathcal{J}}\right)^{-1}. \quad (15)$$

$$\mathbf{R}_\epsilon = \left(\mathbf{\Psi}_{\mathcal{J}}^t\mathbf{\Sigma}_{\mathbf{z}}^{-1}(\rho)\,\mathbf{\Psi}_{\mathcal{J}}\right)^{-1}. \quad (16)$$

Lemma 1 *Let S, K and ρ be fixed and finite . Then as $M \to \infty$,*

$$\mathbf{R}_\epsilon - \frac{M\mathbf{I}_{KS}}{\mathrm{tr}\left[\mathbf{\Sigma}_{\mathbf{z}}^{-1}(\rho)\right]} \xrightarrow[M\to+\infty]{a.s.} 0. \quad (17)$$

$$\sigma_\epsilon^2 \simeq \frac{M}{\mathrm{tr}\left[\mathbf{\Sigma}_{\mathbf{z}}^{-1}(\widehat{\rho})\right]}. \quad (18)$$

3. Performance Analysis

$$\mathcal{R} = \mathcal{C}.\underbrace{\frac{T_c - MT_{ms}}{T_c}}_{\text{Effective Transmission}}$$

$$= \mathcal{C}\left(1 - M\tau\right),$$

$$\mathcal{C}(\Delta) = \log\left(1 + \gamma_{e2e} - \Delta\right)(1 - \mathcal{P}_0)\left(1 - Q\left(\frac{\Delta}{\sigma_\epsilon}\right)\right),$$

$$M = \beta\left(KS + S\log\frac{KN}{S}\right),$$

$$\gamma_{e2e} = \min\left(\frac{\frac{P_s}{N_T}\|\mathbf{f}\|^2}{P_r\rho^2 + N_0}, \gamma_{n^*}\right).$$

4. Numerical Results

Parameter	Value	Parameter	Value
P_s	40 dBm	β	2
P_r	15 dBm	N_0	1
N_T	8 antennas	$\mathcal{P}_0$	0.01
θ	2	K	$\{3,4,5\}$

Table 1: *Simulation parameters*

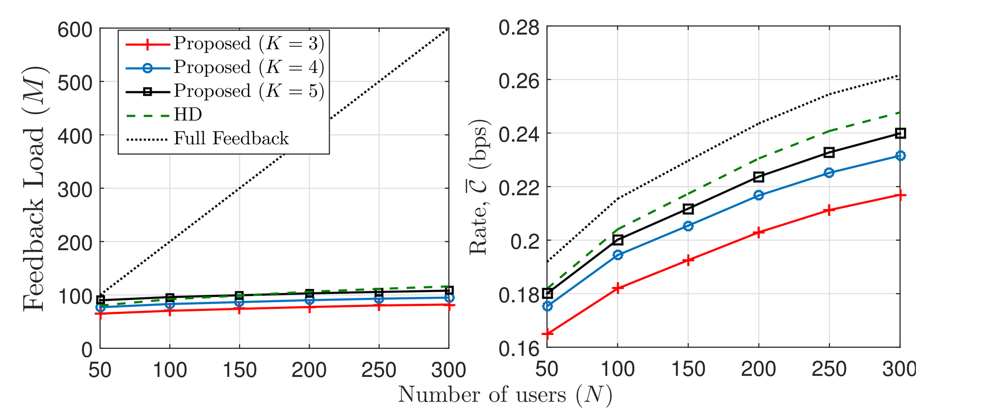

Figure 3: *Ergodic rate and Feedback load as a function of the number of users N.*

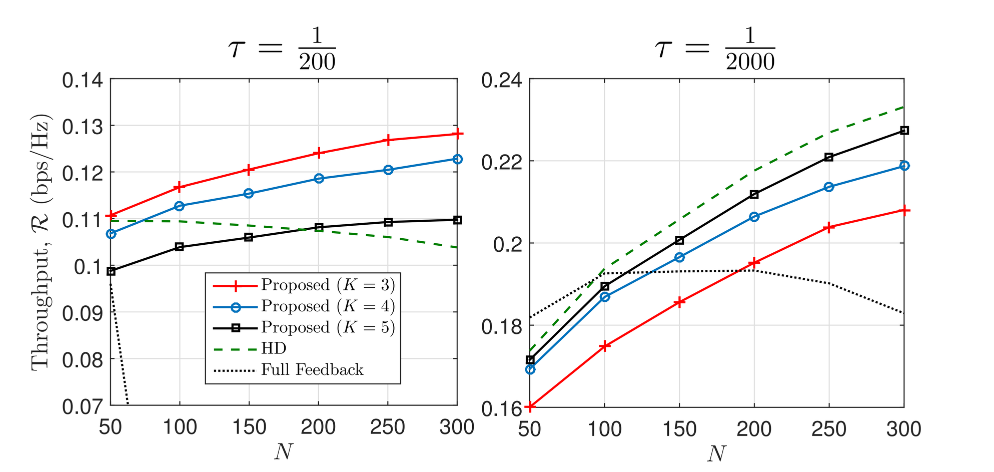

Figure 4: *Achievable throughput as a function of the number of users N.*

5. Conclusion

We proposed a CS-based feedback strategy for user selection in multiuser full duplex relay networks. Based on the theory of compressed sensing, we were able to cast the problem as a block sparse signal recovery and jointly estimate the feedback signal and the loop interference induced by the simultaneous transmission and reception at the relay.

6. References

k. Elkhalil, M. Eltayeb, A. Kammoun, T. Al-Naffouri and H. Bahrami, "On the Feedback Reduction of Relay Aided Multiuser Networks using Compressive Sensing", *IEEE Trans. on Communications*, 2016.